Robust Nonlinear Control of Chemical Reactors

Boris E. Dainson and Daniel R. Lewin
Dept. of Chemical Engineering, Technion IIT, Haifa 32000, Israel

Differential geometry has been found to be an effective tool for controller design for highly nonlinear chemical processes (Kravaris and Chung, 1987; Henson and Seborg, 1990; Daoutidis et al., 1990). Robustness, which remains one of the crucial problems limiting its wider application, has naturally received attention in the literature. Doyle III and Morari (1990) use a conic-sector-based approach to robust controller design, determining bounds on a state-dependent disturbance for which the feedback linearizing control leads to a stable closed-loop system. Kantor and Keenan (1987) formulate sufficient conditions for stability of a control system under input-state linearizing control in terms of bounds on a state-dependent disturbance. The main idea of this method is to apply stability criteria for linear systems under nonlinear state-dependent perturbations to a nominal nonlinear process under linearizing control, and they demonstrate how the choice of controller parameters and the variable transformation influence system robustness. In both of the above approaches, as in many others, the controller is first synthesized on the basis of the nominal model and subsequently modified as necessary to impart robustness.

An alternative approach is to design nonlinear controllers directly for a plant with specified uncertainty. When uncertain parameters enter the model linearly, adaptive control techniques can be incorporated for their estimation, upon which nonlinear controllers can be implemented. Several recent works have been presented based on these ideas with applications to control of chemical reactors (Viel et al., 1995; Dochain et al., 1994). Such approaches are limited to models in which the uncertain parameters appear linearly, and are based on the assumption of constant values of uncertain parameters or their slow variation. Controller design methods exist for systems satisfying different types of matching conditions. The standard matching conditions require that model uncertainty and disturbances belong to the subspace spanned by vectors multiplying control inputs. Such designs with applications from chemical engineering can be found in Kravaris and Palanki (1988) and Arkun and Calvet (1992).

In the approach described herein, the feedback linearizing method itself is modified on the basis of known uncertainty bounds. In this way, the controller synthesis and robustness assurance steps are unified in such a way as to guarantee closed-loop robustness and performance. This is carried out as a straightforward design procedure aimed at fulfilling the design specifications which are formulated as a desired rate of convergence of the regulation error and its derivatives to a window in the state space of a predefined size around a desired trajectory. This is an extension of the method of Slotine and Hedrick (1993) to MIMO systems, both those for which full (input-state) feedback linearization is performed and for those under input-output linearization. Model uncertainty is also allowed in the vectors multiplying the control inputs.

As with any state feedback control design, the proposed method relies on the availability of measurements of all state variables. When this is not the case (such as concentrations that are not easily measured), application of this approach requires the incorporation of state observers. These are often difficult to apply because of the robustness requirements for the overall design, since standard observers (such as extended Kalman filters) are based on an assumption of a perfectly known process model. One type of state observer, the asymptotic observer (Dochain et al., 1992) was developed specifically for stirred tank reactor applications and does not require any knowledge of kinetic expressions. It is based on assumptions of measured temperature and known stoichiometric coefficient matrix, density, specific heat, and heats of reactions. The authors derive a minimal number of required concentration measurements and propose an observer, which provides convergence of estimates of unmeasured concentrations to their true values. Other nonlinear observers with proven robustness properties can also be incorporated, for example, a nonlinear state observer proposed by Gauthier et al. (1992), which can be assigned an arbitrary rate of convergence to the true estimates. Its robustness properties with respect to bounded modeling errors and bounded variance noises are given in Deza and Gauthier (1991). This observer has been successfully used in a previous study (Tartakovsky et al., 1996), but space limitations preclude an illustration of its performance here.

The robust feedback linearizing control design is presented for the case of input-state linearization. The method is then extended to also account for the case of input-output feedback linearization and to uncertainties entering functions multiplying control inputs. Finally, its application on a four-state model describing an exothermic CSTR is demonstrated.

Correspondence concerning this article should be addressed to D. R. Lewin.
Current address of B. E. Dainson: Dept. of Chemical Engineering, City College of New York, 140th St. and Convent Ave., New York, NY 10031.

Robust Feedback Linearization

Equations governing a process are assumed to be in the following standard form

$$\frac{dx}{dt} = f(x) + \sum_{i=1}^{m} g_i(x)u_i$$
$$y = h(x) \tag{1}$$

Consider the case where uncertainty is limited to the term $f(x)$, assumed to be $f(x) = \hat{f}(x) - \tilde{f}(x)$, where $\hat{f}(x)$ describes the nominal system, while $\tilde{f}(x)$ is the model error. Since static state feedback control is applied, it is assumed that the uncertainty does not change the relative degree vector $r = [r_1, \ldots, r_m]$. Later, in the section on model error in $g(x)$, the method will be extended for systems with uncertainty in functions $g_1, \ldots, g_m$. First, the case when input-output linearization yields full (input-state) linearization of the system will be treated: $R_m = n$, where $R_k = \Sigma_{i=1}^{k} r_i$ and n is the system order. Conditions for the implementation of the method to input-output linearization will be given in the section on robust input-output linearization. Following the procedure and notation of Slotine and Hendrick (1993), the transformed state vector is obtained by differentiation of the output vector with respect to the nominal model

$$\begin{matrix} \hat{\mu}_1 = h_1 & \hat{\mu}_2 = L_{\hat{f}} h_1, & \ldots, & \hat{\mu}_{r_1} = L_{\hat{f}}^{r_1 - 1} h_1, \\ \hat{\mu}_{r_1+1} = h_2, & \hat{\mu}_{r_1+2} = L_{\hat{f}} h_2, & \ldots, & \hat{\mu}_{r_2} = L_{\hat{f}}^{r_2 - 1} h_2, \\ \vdots & \vdots & \vdots & \vdots \\ \hat{\mu}_{R_{m-1}+1} = h_m, & \hat{\mu}_{R_{m-1}+2} = L_{\hat{f}} h_m & \ldots, & \hat{\mu}_n = L_{\hat{f}}^{r_m - 1} h_m \end{matrix} \tag{2}$$

Extending Slotine and Hendrick (1993), a new set of variables for the MIMO system is introduced. First, $\overline{\mu}_i$ are defined, which can be thought of as regulation errors and their derivatives according to the nominal model

$$\overline{\mu}_i = \hat{\mu}_i - z_i, \qquad i = 1, \ldots, R_m \tag{3}$$

Exact definition of the so-called synthetic inputs z_i will be given below. Next, the set of s_i is defined

$$s_i = \overline{\mu}_i - \phi_i \,\mathrm{sat}(\overline{\mu}_i/\phi_i), \qquad i = 1, \ldots, R_m \tag{4}$$

where ϕ_i are positive design constants; sat(x) is the saturation function [sat$(x) = x$ if $|x| < 1$, and sat$(x) = $ sign(x), otherwise]. According to the definitions of s_i in Eq. 4, s_i is zero when the corresponding regulation error $\overline{\mu}_i$ is less than ϕ_i. Thus, convergence of all s_i to zero leads to convergence of regulation errors and their derivatives to a hypercube in state space whose size is defined by the values of ϕ_i, which are free design parameters. All the variables defined above rely only on the nominal model. Therefore, they can be explicitly computed if the system state is known. Model uncertainty will enter the following sets of variables

$$\begin{matrix} D_1 = -L_{\tilde{f}} h_1, \; D_2 = -L_{\tilde{f}} L_{\hat{f}} h_1, & \ldots, & D_{r_1} = -L_{\tilde{f}} L_{\hat{f}}^{r_1 - 1} h_1, \\ \vdots & \vdots & \vdots \\ D_{R_{m-1}+1} = -L_{\tilde{f}} h_m, \; D_{R_{m-1}+2} = -L_{\tilde{f}} L_{\hat{f}} h_m & \ldots, & D_{R_m} = -L_{\tilde{f}} L_{\hat{f}}^{r_m - 1} h_m \end{matrix} \tag{5}$$

$$\Delta_i = \begin{cases} D_i + \dot{z}_i^e - \dot{z}_i & \text{if } i = R_k \\ D_i + \dot{z}_i^e - \dot{z}_i + \phi_{i+1} \,\mathrm{sat}(\overline{\mu}_{i+1}/\phi_{i+1}) & \text{for other } i = 1, \ldots, R_m \end{cases} \tag{6}$$

Next, a set of intermediate variables required to derive the control law is defined. These variables are termed as *synthetic inputs* by Slotine and Hendrick

$$\begin{aligned} z_{R_k+1} &= y_{k+1}^r, \qquad k = 0, \ldots, m-1, \qquad R_0 = 0 \\ z_{R_k+2} &= \dot{z}_{R_k+1}^e - F_{R_k+1} \,\mathrm{sat}(\overline{\mu}_{R_k+1}/\phi_{R_k+1}) - \lambda s_{R_k+1}, \\ z_{R_k+3} &= \dot{z}_{R_k+2}^e - F_{R_k+2} \,\mathrm{sat}(\overline{\mu}_{R_k+2}/\phi_{R_k+2}) - \lambda s_{R_k+2} - s_{R_k+1}, \\ &\vdots \\ z_{R_{k+1}} &= \dot{z}_{R_{k+1}-1}^e - F_{R_{k+1}-1} \,\mathrm{sat}(\overline{\mu}_{R_{k+1}-1}/\phi_{R_{k+1}-1}) - \lambda s_{R_{k+1}-1} - s_{R_{k+1}-2} \end{aligned} \tag{7}$$

where λ is a positive constant and y^r is the reference signal. Estimates of the true time derivatives of the system inputs $\dot{z}_i^e$ can be expressed in terms of contributions of the states and inputs, according to the nominal model

$$\dot{z}_i^e = \dot{z}_i^{ex} + \sum_{k=1}^{m} u_k \dot{z}_{ik}^{eu}, \tag{8}$$

where

$$\dot{z}_i^{ex} = L_{\hat{f}} z_i \quad \text{and} \quad \dot{z}_{ik}^{eu} = L_{g_k} z_i.$$

It can now be seen that $\overline{\mu}_i$ is related to regulation errors $(\overline{\mu}_1 = h_1 - y_1^r, \overline{\mu}_{r_1+1} = h_2 - y_2^r, \ldots)$ and their derivatives. The variables D_i and Δ_i are dependent on model uncertainty, and are bounded using the known model error $\tilde{f}(x)$. Only bounds on Δ_i are required, which can be cast as state dependent functions $P_i(x)$, such that

$$|\Delta_i| \leq P_i(x) \tag{9}$$

Finally, the following control law is proposed

$$u = [H(x) - \dot{z}_R^{eu}]^{-1}\{-L_f^r h + \dot{z}_R^{ex} - [P\,\text{sat}(\tilde{\mu}/\phi)]_R - \lambda s_R - s_{R-1}\} \tag{10}$$

where

$$H_{kl}(x) = L_{g_l} L_f^{r_k-1} h_k, \qquad L_f^r h = \left[L_f^{r_1} h_1, \ldots, L_f^{r_m} h_m\right]^T \tag{11}$$

and the subscript R corresponds to the vector: $(\cdot)_R = [(\cdot)_{R_1}, \ldots, (\cdot)_{R_m}]^T$. This controller ensures exponential decrease of the Lyapunov function

$$V = \frac{1}{2}\sum_{i=1}^{R_m} s_i^2, \tag{12}$$

since by straightforward but rather cumbersome computations, it can be verified that

$$\frac{dV}{dt} + 2\lambda V = \sum_{i=1}^{n} s_i[\Delta_i - P_i\,\text{sign}(s_i)] \tag{13}$$

where each term in the righthand side summation is negative semidefinite as a result of the definitions of P_i (Eq. 9). This is enough to show that the regulation error is asymptotically bounded by appropriate constants ϕ_i and the complete state vector remains bounded. The matrix H to be inverted in the definition of the control law (Eq. 10) is nonsingular if the assumption of a common relative degree vector for all admissible submodels is satisfied. The values of the design parameters can be chosen to achieve desired performance specifications; faster convergence is attained by increasing the value of λ, while tighter set point tracking can be achieved by reducing ϕ_i. These performance improvements will of course be attained at the price of stronger control action. It should be noted that the proposed approach is not, strictly speaking, a feedback linearizing control because it does not result in a linear closed-loop relationship. Application of the method to control of a series of CSTRs and its comparison with a standard feedback linearization controller with tunable parameters is given in Dainson and Lewin (1995).

Robust Input-Output Linearization

The described robust linearization technique also can be applied to systems which do not satisfy conditions for full input-state linearization or when the system outputs are defined such that it is not achieved by the standard input-output linearization ($R_m < n$). The main problem which arises in such a case is to ensure boundness of $n - R_m$ additional components of the new state vector $\hat{\mu}$ needed to complete the state transformation, because these components are not involved in the Lyapunov function (Eq. 12). If the distribution spanned by the vector fields $g_1, \ldots, g_m$ is involutive, the state transformation (Eq. 2) can be completed such that the zero dynamics will be independent of the input (see, for example, Isidori, 1995)

$$\frac{d\eta}{dt} = p(\hat{\mu}, \eta) \tag{14}$$

For any fixed value of $\hat{\mu}$, Eq. 14 is termed the zero dynamics of the system. The control law (Eq. 10) yields exponential convergence of the s vector to zero, which ensures that $\hat{\mu}$ is bounded for bounded and sufficiently smooth reference signals. If the zero dynamics are bounded-input bounded-output (BIBO) with $\hat{\mu}$ being the input and η considered as the output, the proposed control will preserve all the state of the system bounded.

Model Error in $g(x)$

Here, the control law (Eq. 10) is modified to ensure convergence of the Lyapunov function when the function $g(x)$ is not exactly known but can be expressed as the difference between a nominal part and a term representing model error: $g(x) = \hat{g}(x) - \tilde{g}(x)$. The control law is now defined as $u = \hat{u} + \bar{u}$, where the expression for $\hat{u}$ is taken as in the case of exactly known $g(x)$ (from Eq. 10). In this case, the Lyapuňov function transient is described by

$$\frac{dV}{dt} + 2\lambda V = \sum_{i=1}^{n} s_i[\Delta_i - P_i\,\text{sat}(\bar{\mu}_i/\phi_i)] - s_R^T \Delta H \hat{u} + s_R^T (H - \Delta H)\bar{u} \tag{15}$$

where

$$\Delta H_{kl}(x) = L_{\tilde{g}l} L_f^{r_k-1} h_k - L_{\tilde{g}_l} Z_{R_k l} \tag{16}$$

and $H(x)$ is defined similarly to Eq. 11 with nominal expressions of $g(x)$ substituting their true values

$$H_{kl}(x) = L_{\hat{g}_l} L_f^{r_k-1} h_k \tag{17}$$

Since it was shown that the first term of the righthand side of Eq. 15 is negative semidefinite, it is enough to select $\bar{u}$ that will provide nonpositivity of the rest of the expression. Taking $\bar{u} = -\gamma H^{-1} s_R$, with γ being a positive constant to be determined later, this obtains

$$s_R^T(H - \Delta H)\bar{u} = -\gamma(s_R^T s_R + s_R^T \Delta H H^{-1} s_R) \leq -\gamma|s_R|^2[1 - \bar{\sigma}(\Delta H)\bar{\sigma}(H^{-1})] = -\gamma|s_R|^2[1 - \bar{\sigma}(\Delta H)/\underline{\sigma}(H)] \tag{18}$$

State dependent bounds will now be introduced on the uncertainty related with $g(x)$ and, for convenience, on $H(x)$

$$\bar{\sigma}[\Delta H(x)] \leq \alpha(x); \qquad \underline{\sigma}[H(x)] \geq \beta(x) > 0 \tag{19}$$

Substituting Eqs. 16–19 into Eq. 15 gives

$$\frac{dV}{dt}+2\lambda V\le |s_R||\hat{u}|\alpha-\gamma|s_R|^2[1-\alpha/\beta] \tag{20}$$

The uncertainty in $g(x)$ should satisfy the condition which can be interpreted as less than 100% gain uncertainty.

$$\alpha(x)/\beta(x)<1. \tag{21}$$

For MIMO systems, input and output scaling influences the ratio $\alpha(x)/\beta(x)$. As will be demonstrated in the example below, an appropriate scaling might be necessary to meet the above condition. If this is satisfied, it is enough to take $\gamma = |\hat{u}|\alpha/|s_R|(1-\alpha/\beta)$ for $|s_R|\neq 0$ and any value of γ otherwise to ensure that the lefthand-side of Eq. 18 is negative semidefinite. The expression for the additional control term to treat uncertainty in $g(x)$ takes the following form

$$\bar{u}=-\frac{|\hat{u}|\alpha}{|s_R|(1-\alpha/\beta)}H^{-1}s_R \quad \text{for} \quad |s_R|\neq 0$$

$$\text{and any other value of } \bar{u} \text{ otherwise} \tag{22}$$

It should be noted that the term $\bar{u}$, as defined in Eq. 22, is a bounded but generally discontinuous function of the state in the vicinity of $|s_R|=0$. For the SISO case, this definition can easily be modified to generate a continuous function

$$\bar{u}=-\frac{|\hat{u}|\alpha}{(1-\alpha/\beta)H}\,\mathrm{sat}(\bar{\mu}_R/\phi_R) \tag{23}$$

Example

An exothermic CSTR described by Allgöwer and Ilchmann (1995) is considered. The target product P is produced from the feed species A according to the following reaction scheme

$$A\xrightarrow{r_1}B\xrightarrow{r_2}P, \qquad B\xrightarrow{r_3}X$$

The model describing the reactor can be written as

$$\frac{dc_A}{dt}=\frac{F}{\nu}(c_{Ao}-c_A)-k_1(T)\cdot c_A \tag{24}$$

$$\frac{dc_B}{dt}=-\frac{F}{\nu}c_B+k_1(T)\cdot c_A-k_2(T)\cdot c_B-k_3(T)\cdot c_B \tag{25}$$

$$\frac{dc_P}{dt}=-\frac{F}{\nu}c_P+k_2(T)\cdot c_B \tag{26}$$

$$\frac{dT}{dt}=\frac{F}{\nu}(T_0-T)+\Delta h_1\cdot k_1(T)\cdot c_A+\Delta h_2\cdot k_2(T)\cdot c_B$$

$$+\Delta h_3\cdot k_3(T)\cdot c_B+\frac{j_Q}{\nu\cdot\rho\cdot c} \tag{27}$$

The temperature of the reactor T and the product concentration c_P are controlled by the coolant flow j_Q and the reactant feed rate F. The reaction coefficients are given in Arrhenius form: $k_i(T)=k_{i0}\cdot\exp(E_i/T)$, $i=1, 2, 3$, with parameter values given in Allgöwer and Ilchmann (1995).

This system is affine with a well-defined relative degree vector $r=[1\ 1]^T$, which is invariant with uncertainty for nonvanishing values of c_p and a finite volume of the reactor ν. The described approach is not only limited to parametric uncertainty: any functional bounds on kinetic expressions, heats of reactions, and/or heat-transfer laws which preserve the vector relative degree are allowed. For demonstration purposes, model uncertainty will be assumed as being due to independent variations of the k_{i0} $(i=1, 2, 3)$ coefficients by no more than 30% of their nominal values. It is possible to find a coordinate transformation of the model equations that makes the zero dynamics independent of the inputs, since it is easy to check that the distribution spanned by the vector fields

$$g_1=[(c_{Ao}-c_A),\quad -c_B,\quad -c_P,\quad (T_0-T)]^T/V$$

$$\text{and}\quad g_2=[0,\quad 0,\quad 0,\quad 1]^T/(V\cdot\rho\cdot c) \tag{28}$$

is involutive. One of the possible transformations has the following form

$$\eta_1=\ln(c_{Ao}-c_A)+\ln c_B \qquad \eta_2=\ln(c_{Ao}-c_A)+\ln c_P \tag{29}$$

The zero dynamics in the original coordinates are given by Eqs. 24 and 25, for c_A and c_B, where F, c_P and T are constant at their steady-state values. These constitute a stable linear system, implying asymptotically stabilizing global input/output linearization. It is easy to verify that any state transformation which is globally Lipschitz and has a globally Lipschitz inverse will preserve this property. The state transformation given above does not satisfy these properties globally. However, in the region $-M\le c_A\le c_{Ao}-\epsilon$ and $\epsilon\le c_B\le M$, where ϵ and M are arbitrary positive constants, this transformation is Lipschitz with a Lipschitz inverse. This is enough to show that the zero dynamics have a wide enough region of exponential attraction which is larger than the operating region (note that only positive concentrations are of interest and $c_A<c_{Ao}$). This does not formally satisfy requirements for global stability but is enough to prove local L_p stability of the control algorithm and to expect its stability in the operating region.

Since concentrations and temperature have significantly different scales of typical variations, output scaling is desirable to achieve comparable speeds of convergence. Thus, the outputs are selected as $y_1=p_1\cdot c_p$, $y_2=p_2\cdot T$ with $p_1=20$ and $p_2=1$. The desired control tolerances are ± 0.02 mol/L for concentration of P and ± 1 K for temperature. This defines two controller parameters (after taking scaling into account): $\phi_1=0.4$ and $\phi_2=1$. The value for λ, which determines rate of convergence, is chosen as $\lambda=1.7$ min^{-1}. Increasing λ speeds up the response at the price of more aggressive control. Applying the robust feedback linearization procedure (Eqs. 2–9)

$$\Delta_1 = D_1 = p_1 \cdot \tilde{k}_2(T) \cdot c_B \quad (30)$$

$$\Delta_2 = D_2 = -\frac{p_2}{V \cdot \rho \cdot c}[\tilde{k}_1(T) \cdot c_A \cdot \Delta h_1 + \tilde{k}_2(T) \cdot c_B \cdot \Delta h_2 + \tilde{k}_3(T) \cdot c_B \cdot \Delta h_3] \quad (31)$$

Functions $P_i(x)$ that are bounds on D_i are easy to find

$$P_1 = p_1 \cdot \tilde{k}_2^{\max}(T) \cdot c_B \quad (32)$$

$$P_2 = \frac{p_2}{V \cdot \rho \cdot c}[\tilde{k}_1^{\max}(T) \cdot c_A|\Delta h_1| + \tilde{k}_2^{\max}(T) \cdot c_B|\Delta h_2| + \tilde{k}_3^{\max}(T) \cdot c_B|\Delta h_3|] \quad (33)$$

Here $\hat{k}_i(T)$, $\tilde{k}_i(T)$ and $\tilde{k}_i^{\max}(T)$ are the nominal kinetic coefficients, their errors and the maximal allowed magnitudes of the errors, respectively (for $i = 1, 2, 3$).

The proposed method also can be applied to problems of disturbance rejection. Unmeasured disturbances can be considered as unknown (and time varying) model parameters. If their variations have known bounds, such a system will satisfy the examined problem formulation. Allgöwer and Ilchmann (1995) consider c_{A0} and T_o as disturbances. Noting that in Eq. 27, T_o multiplies one of the manipulated inputs, and a disturbance in T_o ($T_o = \hat{T}_o - \tilde{T}_o$ with $|\tilde{T}_o| \leq \tilde{T}_o^{\max}$) is equivalent to uncertainty in $g(x)$ in the generalized nonlinear formulation (Eq. 1). For this example, it is required that regulatory control should be able to adequately reject the disturbance $\tilde{c}_{A0} = -3$ mol/L and $\tilde{T}_o = 10$ K. Scaling of inputs will be necessary to satisfy the condition (Eq. 21). The following notation is introduced: $j_Q = w_1 u_1$ and $F = w_2 u_2$ where appropriate scaling factors w_1 and w_2 are selected. This input scaling does not influence Eqs. 30–33, but appears in the control law, which now takes the following form

$$\hat{u} = H(x)^{-1}\begin{bmatrix} -L_f y_1 - P_1 \operatorname{sat}(\tilde{\mu}_1/\phi_1) - \lambda s_1 \\ -L_f y_2 - P_2 \operatorname{sat}(\tilde{\mu}_2/\phi_2) - \lambda s_2 \end{bmatrix} \quad (34)$$

with

$$H(x) = \begin{bmatrix} -p_1 \cdot w_1 \cdot c_P & 0 \\ p_2 \cdot w_1(\hat{T}_0 - T)/\nu & p_2 \cdot w_2/(\rho \cdot \nu \cdot c) \end{bmatrix}$$

$$\Delta H(x) = \begin{bmatrix} 0 & 0 \\ p_2 \cdot w_1 \cdot \tilde{T}_0/\nu & 0 \end{bmatrix} \quad (35)$$

The choice of $\alpha(x)$ is obvious: $\alpha(x) = p_2 \cdot w_1 \cdot \tilde{T}_0^{\max}/\nu$. The condition (Eq. 21) is not satisfied globally for the matrices given by Eq. 35. This is similar to the problem discussed previously, where it was not possible to guarantee global exponential stability of the zero dynamics. However, as before, convergence of the control algorithm can be expected if this condition is met in a wide enough operating region. Strict extension of the proposed technique to control in bounded regions requires additional attention, but is facilitated by knowledge of the Lyapunov function (Eq. 12) with known convergence properties (Eq. 13). The following parameter values were chosen: $w_1 = 1$, $w_2 = 20$, and $\beta(x) \equiv \sigma[H(x)]$ which is easily computed on-line. This satisfies the condition (Eq. 21) for $\tilde{T}_0^{\max} = 10$ K in the region given by $c_P > 0.8$ mol/L and $323 \text{ K} \leq T \leq 364$ K.

A typical run of the system under the proposed control law, showing its convergence from an initial point to the set point $c_p = 1$ mol/L, $T = 353.15$ K is shown in Figure 1. Each plot is composed of the eight responses corresponding to the extreme k_{i0} values on the uncertainty cube. Note that the offset satisfies the predefined control tolerances. Figure 1 shows fast convergence to the desired set point without significant response to the disturbance, which compares favorably with the responses presented in Allgöwer and Ilchmann

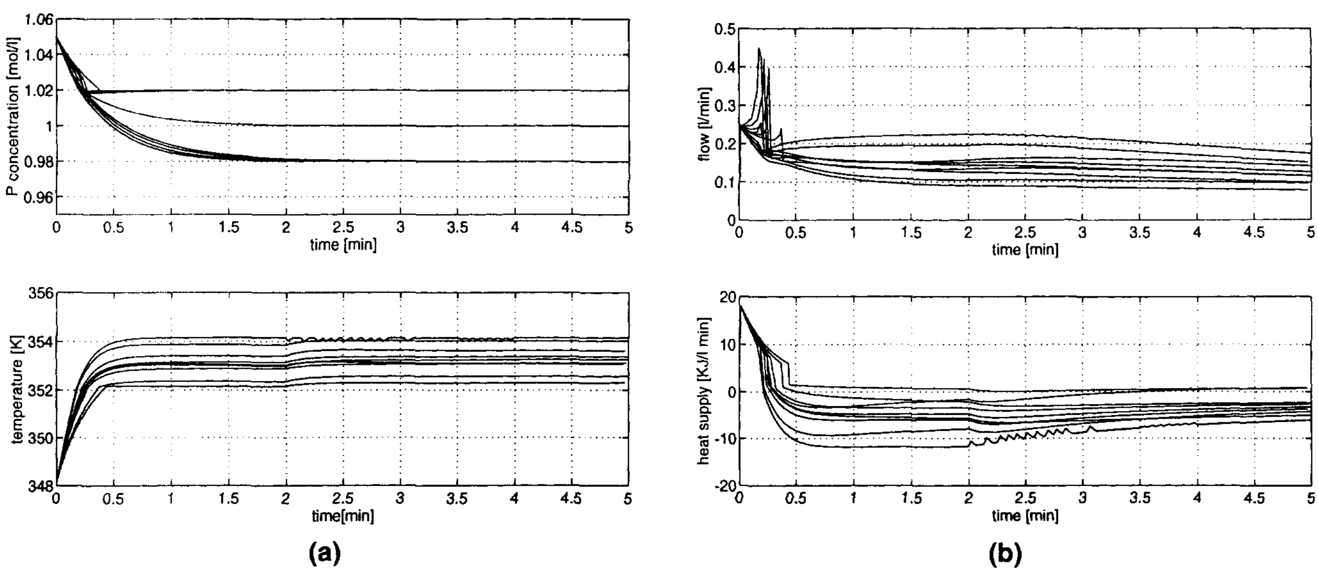

Figure 1. Closed-loop response to a set point step change, followed by a step change in the disturbance (at $t = 2$).
(a) Response of outputs; (b) response of manipulated variables.

(1995). Despite the disturbance at $t = 2$ min, the output trajectories stay inside the desired control tolerance envelope. It should be noted that disturbances in c_{A0} have no impact on the closed-loop system design and its performance, since the relative degree of this disturbance is $[2\ 2]^T$, which is higher than that of the manipulated inputs. This disturbance effects the outputs only through c_A, a measured state variable, which is completely compensated by the term $-L_f^r h$ in the control law (Eq. 10). A chattering-like behavior is observed on the upper curve for temperature in Figure 1a and on the corresponding lower curve for the heat supply in Figure 1b. This can happen when any of the regulation errors is at its boundary value ϕ_i and is due to the discontinuity of the control law at $s_i = 0$ mentioned above. The tracking error is reduced by selecting smaller values for ϕ_i. More delicate tuning can be achieved by selecting different values of λ for each output.

Conclusions

To account for model uncertainty, a robust feedback linearizing control design has been proposed for a general MIMO nonlinear system. This has been demonstrated on a four-state CSTR problem on which robust input-output linearization is performed. The described method can also be applied in cases where uncertainty is defined in a functional form, and allows for infinitely fast variation of uncertain terms within predefined bounds. These features are totally different from standard requirements imposed by adaptive control methods, which are often limited to models in which the parameters enter linearly. Such properties of the method allow for treatment of unmeasured and possibly fast-varying disturbances as bounded uncertain parameters as has been demonstrated in the example. The method allows any desired control tolerance and rate of convergence to be attained at the price of large and fast varying control actions.

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